INSTITUT PENYELIDIKAN MATEMATIK

OLIMPIAD MATEMATIK UNIVERSITI MALAYSIA 2024
OMUM2024

PEPERIKSAAN PERINGKAT SARINGAN

Tarikh	:	27 April 2024
Masa	:	9.30 am – 12.30 pm
Tempoh	:	3 jam

Arahan kepada calon:

1. Jawab **SEMUA** soalan.
2. Kalkulator adalah **TIDAK** dibenarkan sepanjang peperiksaan berlangsung.
3. Soalan adalah dalam bahasa Inggeris.
4. Markah diberi untuk jalan kerja dan jawapan yang tepat.

Question 1

Determine all real numbers x such that

$$\sqrt{x+7-4\sqrt{x+3}}+\sqrt{x+28-10\sqrt{x+3}}=3.$$

[10 marks]

Question 2

On the interval of $[0,\pi)$, solve the equation $\cos 2\theta+\sin\ \theta=0$.

Hint: $\cos 2\theta=1-\ 2\,sin^2\theta$.

[10 marks]

Question 3

Write down the number $\frac{\sqrt[3]{2}}{1+\sqrt[3]{2}+3\sqrt[3]{4}}$ as a fraction with integer denominator.

[10 marks]

Question 4

Given a cubic function

$$p(X)=\ A^2X^3-X-8$$

where $A\in\mathbb{R}$ and $A\neq 0$. Is it possible for this cubic function has three roots in the open interval $(0,1)$? If not, find the possible maximum root(s) that $p(X)$ can has in the open interval $(0,1)$. Hence, find the values of A.

[10 marks]

Question 5

Let $a_0, a_1, \cdots, a_n$ be real numbers such that

$$a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{a_n}{n+1} = 0.$$

Prove that

$$a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n = 0$$

has at least one solution in the interval $(0, 1)$.

[10 marks]

Question 6

Find all $f: \mathbb{R} \to \mathbb{R}$ such that for all $x, y \in \mathbb{R}$, $f(x) + f(y) = f(x + y)$ and $f(x^{2024}) = f^{2024}(x)$.

[10 marks]